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SPECIAL BRIEF NOTE

BOUNDARY LAYERS EXCITED BY LOW FREQUENCY DISTURBANCES—KLEBANOFF MODE

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The behaviour of wall-bounded shear layers that can be represented by a Blasius profile is studied with reference to very low frequency excitation. The whole shear layer executes a heaving motion, which is referred to as the Klebanoff mode. This has been observed experimentally for a long time, but no theoretical explanation was available so far. Here some theoretical results are presented which indicate that such low frequency excitation produces three-dimensional disturbance waves whose wave lengths are very large compared to the shear-layer thickness. This, and other properties of such a wave system, explains the observed Klebanoff mode.

1. INTRODUCTION

TRANSITION from laminar to turbulent flow is associated initially with spatio-temporal growth of disturbances in the shear layer known as Tollmien–Schlichting (TS) waves (Schlichting 1977). TS waves excited by moderate frequency sources have wavelengths comparable to the shear-layer thickness. However, the situation is qualitatively different for low frequency excitations, as shown in Gaster *et al.* (1994), who investigated the flow field due to a shallow oscillating bump, where the frequency of oscillation is very low (2 Hz). For this case no instability waves were observed; instead, the whole boundary layer executed heaving motion. It is worth mentioning that similar such experiments were performed earlier by Taylor (1939) and Klebanoff (unpublished). Klebanoff called this the breathing mode of motion. There is a resurgence of interest in this problem in recent times, and it is now referred to in the literature as the Klebanoff mode.

There are two noticeable features of the experiments reported by Gaster *et al.* (1994). Firstly, the mean flow field is adequately represented by the Blasius profile and, for the parameter ranges applicable, the two-dimensional instability studies do not reveal the presence of any eigen-solutions that decay as one approaches the edge of shear layer. Secondly, the disturbance field is three-dimensional but it propagates predominantly in the streamwise direction. It is easy to see that if the disturbance fields have very large wavelengths then the experimental results will only indicate a heaving motion for a *small* test-section length–as in all the reported experiments. In Gaster *et al.* (1994) the measurement stations were located at 70 and 105 boundary layer thicknesses and *were therefore expected to be in the far-field of the oscillating bump*. Unlike the receptivity analysis of Sengupta *et al.* (1994), one can perform the usual stability analysis to study the far field.

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The interesting aspect of the Klebanoff mode of motion is that although the mean flow is two-dimensional, the flow does not support a two-dimensional disturbance field. This has led Gaster *et al.* (1994) to comment that a *proper mathematical account of these disturbances has not yet appeared*. Here we show for the first time that while two-dimensional disturbance waves are not supported, three-dimensional disturbances are valid solutions, with the same qualitative features as the experimentally observed ones.

2. FORMULATION

We present the formulation with reference to a three-dimensional disturbance field for two-dimensional mean flow, along with a parallel flow approximation. As the generated waves evolve spatially, it is not sufficient to investigate the two-dimensional disturbance field and invoke Squire's theorem. The disturbance field is characterized by the following disturbance normal velocity:

$$v'(x, y, z, t) = \frac{1}{4\pi^2} \iint_{\mathrm{Br}} \phi(y, \omega_0; \alpha, \beta) \mathrm{e}^{i(\alpha x + bz - \omega_0 t)} \, \mathrm{d}\alpha \, \mathrm{d}\beta \tag{1}$$

with α and β as the streamwise and spanwise wavenumber, ω_0 the circular frequency of excitation, and Br the line along which the Bromwich integral is performed int he complex plane.

In any experiment conducted in a closed wind tunnel, the spanwise wavenumber will have a lower cut-off (β_0) determined by the spanwise extent of the tunnel, i.e.

$$\beta_0 = 2\pi/\lambda_z$$

where the spanwise wavelength is twice the tunnel width. For such cases

$$v'(x, y, z, t) = \frac{1}{2\pi} \sum_{n=1}^{\infty} \int_{\mathrm{Br}} \phi(y, \omega_0, \alpha, n\beta_0) \mathrm{e}^{i(\alpha x + n\beta_0 z - \omega_0 t)} \, \mathrm{d}\alpha \tag{2}$$

The Fourier-Laplace transform ϕ in equation (2) is obtained from the solution of the Orr-Sommerfeld equation given by

$$\begin{aligned} &\phi^{\mathrm{iv}} - 2(\alpha^2 + n^2\beta_0^2)\phi'' + (\alpha^2 + n^2\beta_0^2)^2 \phi \\ &\qquad = \mathrm{i}\,\mathrm{Re}\{(\alpha U + n\beta_0 W - \omega_0)[\phi'' - (\alpha^2 + n^2\beta_0^2)\phi] - (\alpha U'' + n\beta_0 W'')\phi\} \end{aligned} \tag{3}$$

In equation (3) $U(y)$ and $W(y)$ are the parallel mean flow. Re is the Reynolds number based on the displacement thickness of the boundary layer. The present investigation shows that for the very low frequency excitation two-dimensional modes cease to exist for the Blasius boundary layer, and hence we have investigated the three-dimensional flow field. For a given Reynolds number and excitation frequency, the streamwise wavenumber has been located here as an eigenvalue after fixing the spanwise wave number. The spanwise wave number has been decided based on the experimental conditions of Gaster *et al.* (1994). The eigenvalues are located using an extension of the compound matrix method following the general methodology given in Sengupta *et al.* (1994).

3. RESULTS AND DISCUSSION

For all the calculations we have used the compound matrix method in double precision, taking 1600 uniformly distributed points for a maximum Blasius coordinate of $\eta = 12$. While the method of Mack (1976) is used for approximately locating the eigenvalues, these have been located exactly by an eigenvalue finder. The accuracy of the method has already been demonstrated in Sengupta *et al.* (1994).

Some details of the experimental conditions of Gaster *et al.* (1994) are given briefly. A flat plate was mounted in the test-section of a low disturbance wind tunnel which is 3·5 m long and 0·91 m by 0·91 m in cross-section. The three-dimensional velocity field was created by a circular bump of 20 mm diameter which was located 400 mm from the leading edge of the plate. At the location of the bump the undisturbed boundary layer had a thickness of $\delta^* = 0{\cdot}99$ mm. Based on this thickness and free-stream speed of 18·10 m/s the Reynolds number is found to be 1196. The circular bump was oscillated at a frequency of 2 Hz, which makes the non-dimensional circular frequency (ω_0) equal to $6{\cdot}248 \times 10^{-4}$. The span of the test-section is 920 (approximately) times δ^*, and the largest spanwise wavelength that can be supported is twice this. This is the rationale for fixing the lower limit of spanwise wave-number, β_0, and the corresponding value is $3{\cdot}41777 \times 10^{-3}$. From Figure 11 of Gaster *et al.* (1994) one sees that most of the energy is carried by only the first ten modes—where these modes in Gaster *et al.* (1994) are different from the fundamental and its harmonics used in the present formulation. This corresponds to $n\beta_0 = 0{\cdot}7854$—a value up to which the present investigation is extended.

It is seen that the two-dimensional modes disappear as the circular frequency is reduced below a critical level. To show the relative roles of various modes at all frequencies, the real and imaginary part of the wavenumbers (α_r, α_i) are plotted in Figure 1(a,b) against the nondimensional frequency for a Reynolds number of 1000. The three modes shown are the same as those that were identified and tabulated in Sengupta *et al.* (1994). The associated phase speed and group velocity of these modes are shown in Figure 2(a,b). All the modes shown here display discontinuous behaviour as the frequency is decreased. The reason for this discontinuous behaviour is not understood, and it deserves further attention.

If the fluid dynamical system is excited at a frequency less than this frequency (ω_d) we obtain a negative value of α_r for which the group velocity is negative. Thus these waves at frequencies less than ω_d travel upstream. The corresponding high negative values of α_i indicate that these waves are highly damped. Thus it is seen that there are no downstream-propagating discrete eigen-solutions for two-dimensional forcing, for a frequency value less than 0·0026 for Re = 100 or 0·0022 for Re = 1196.

This prompted us to investigate whether the Blasius profile supports three-dimensional downstream-propagating spatial modes. The spatial eigenvalues are located in the same manner as was done in Sengupta *et al.* (1994) following the method due to Mack (1976). Once the eigenvalues are located, the streamwise and spanwise component of the group velocity, V_g, is obtained numerically from the following expression:

$$V_g = \left(\frac{\partial \omega}{\partial \omega_r}, \frac{\partial \omega}{\partial \beta_r}\right) \tag{4}$$

The numerical evaluation of the components of group velocity requires three eigenvalue evaluations. The wavenumber, damping rate, phase speed and x and z components of the group velocity are given in Figures 3(a,b) and 4(a,b,c) as a function of the spanwise

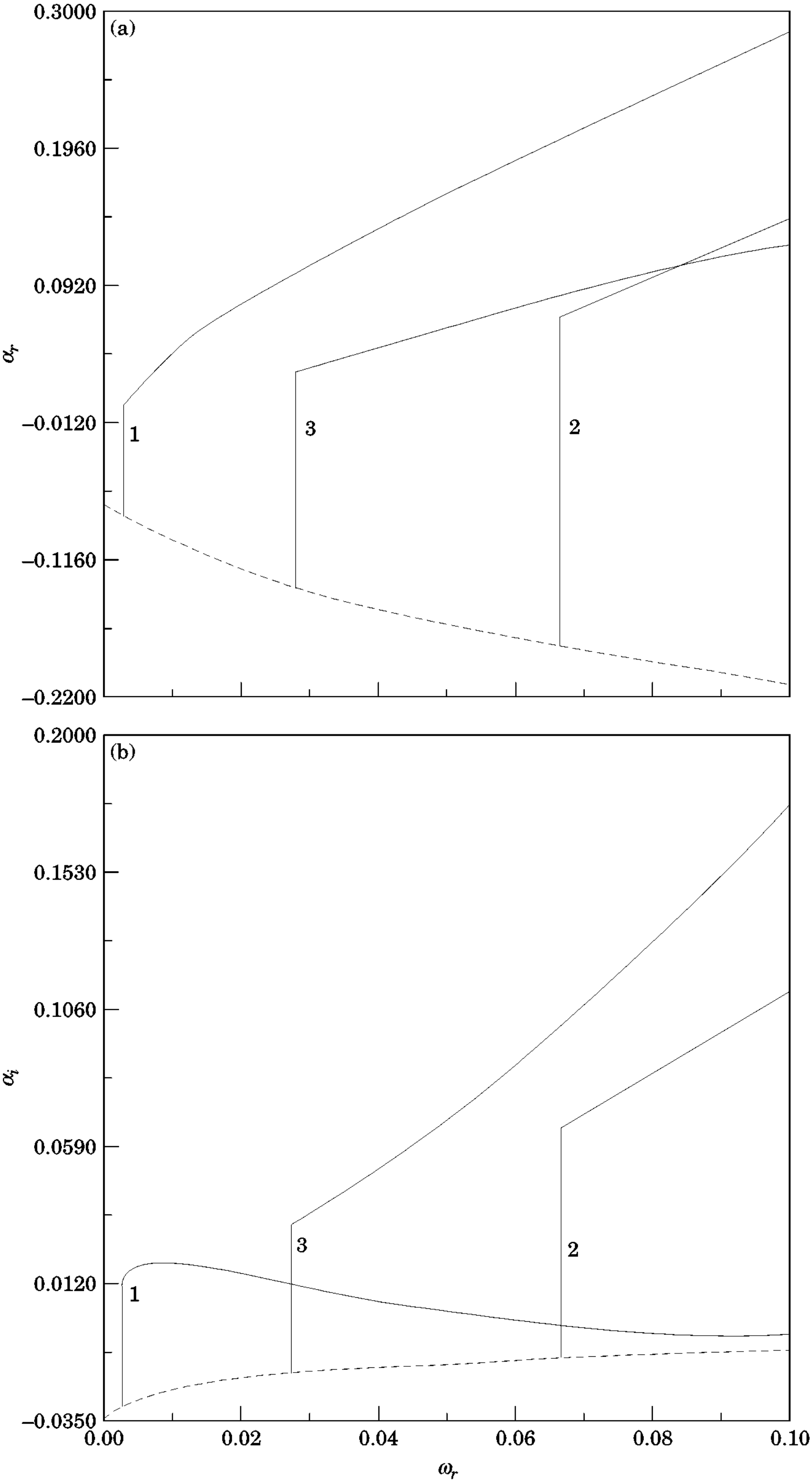

Figure 1. (a) Real wavenumber of three distinct eigenmodes plotted against frequency of excitation at Re = 1000. (b) Imaginary wavenumber of three distinct eigenmodes plotted against frequency of excitation at Re = 1000. (1) TS mode; (2) second mode; (3) third mode. Dotted curve corresponds to upstream propagating mode.

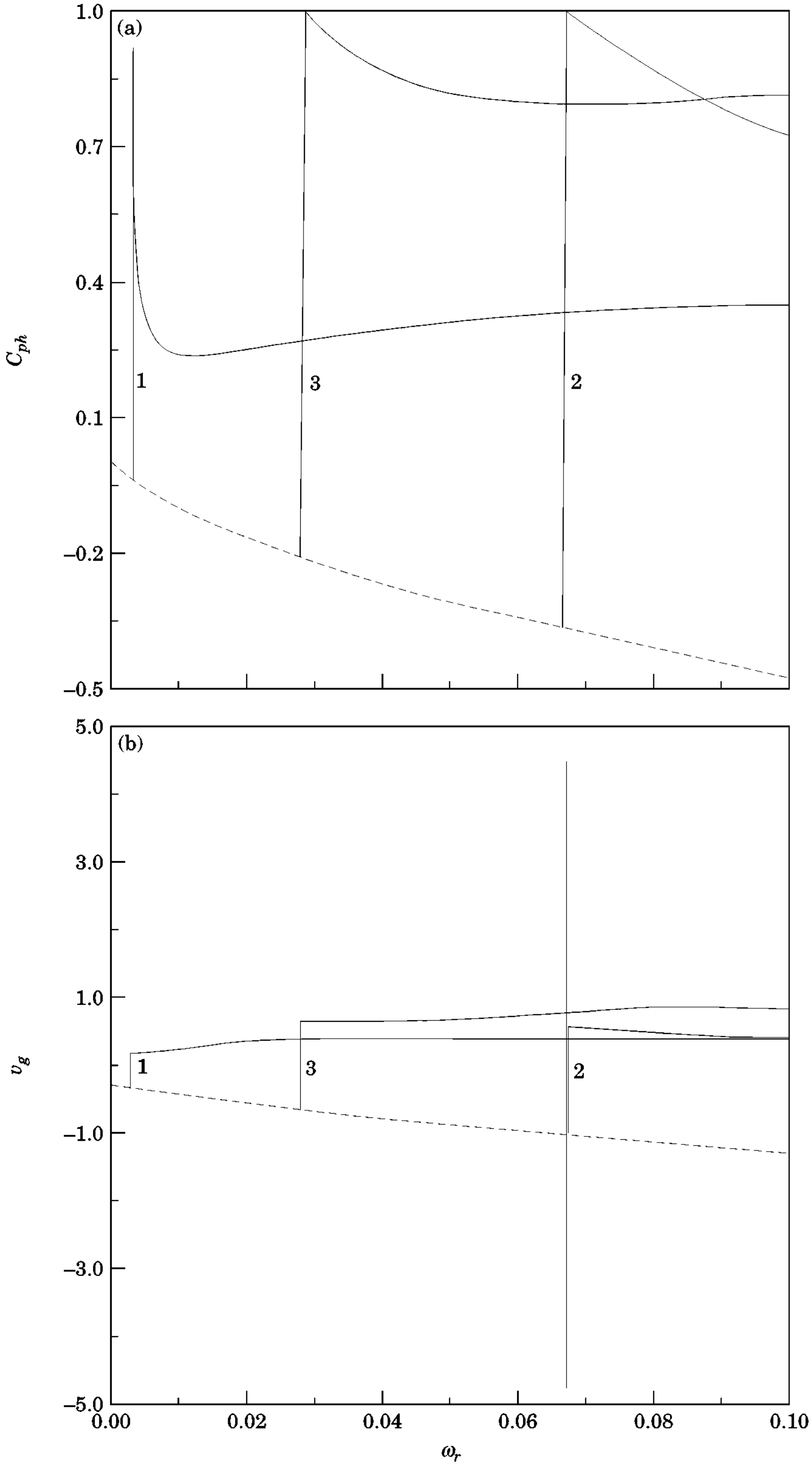

Figure 2. (a) Phase speed of three distinct eigenmodes plotted against frequency of excitation at Re = 1000. (b) Group velocity of three distinct eigenmodes plotted against frequency of excitation at Re = 1000. (1) TS mode; (2) second mode; (3) third mode. Dotted curve corresponds with upstream propagating mode.

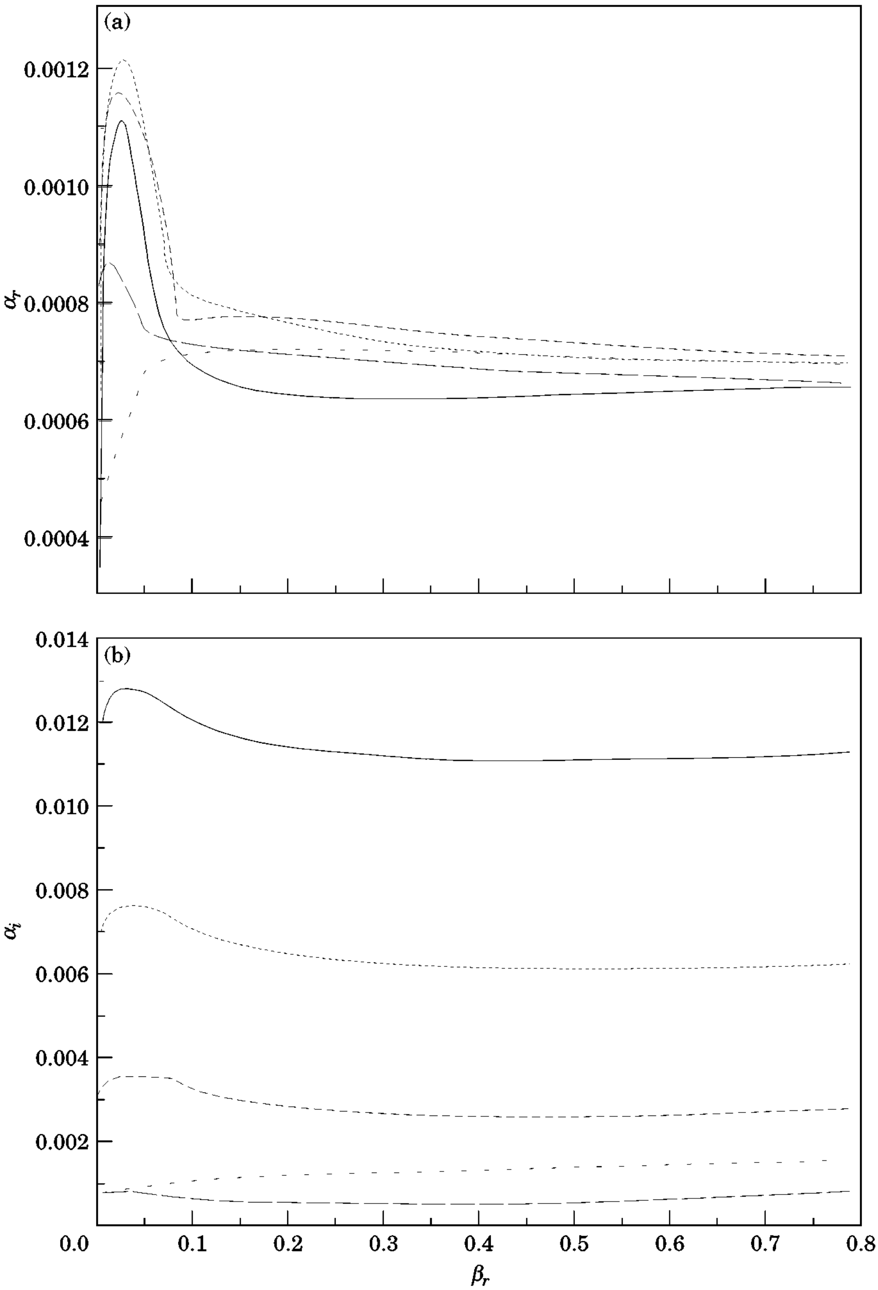

Figure 3. (a) Streamwise wavenumbers of the detected modes as a function of spanwise wavenumber. (b) Streamwise decay rates of the detected modes as a function of spanwise wavenumber. —, Mode 1; · · · · · · , mode 2; - - - -, mode 3; ······,mode 4; — —, mode 5.

wavenumber for the detected modes. It is clearly evident that all the excited modes are damped. Far away from the exciter, only the effect of the least damped modes (fourth and fifth) will be recorded. Here the least damped modes have wavelengths of the order of 10^4 times δ^*. The experiments of Gaster *et al.* (1994) measured only up to 500 δ^* downstream of

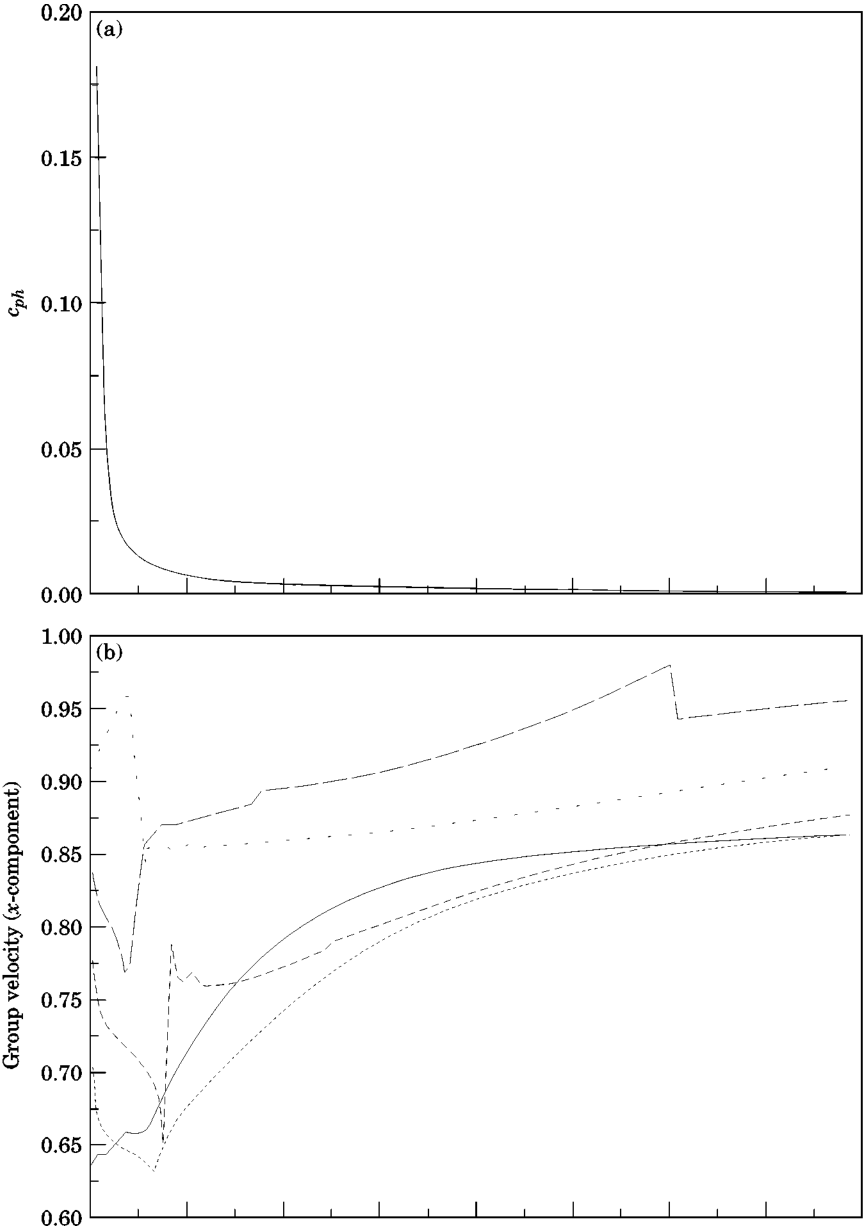

Figure 4. (a) Phase speeds of the detected modes as a function of spanwise wavenumber. (b) Streamwise component of group velocities of the detected modes as a function of spanwise wavenumber. (c) Cross flow component of group velocities of the detected modes as a function of spanwise wavenumber. —, Mode 1; · · · · · , mode 2; - - - - , mode 3; ······, mode 4; — —, mode 5.

the exciter, and hence it appeared that the whole boundary layer was heaving. Also, the directions of disturbance propagation of the fourth and fifth modes are predominantly in the x-direction, since the z-component of the group velocity is negligibly small.

For increasing spanwise wavenumbers, one can see that the last two modes are the least damped ones and these are dominant in describing the disturbance field away from the

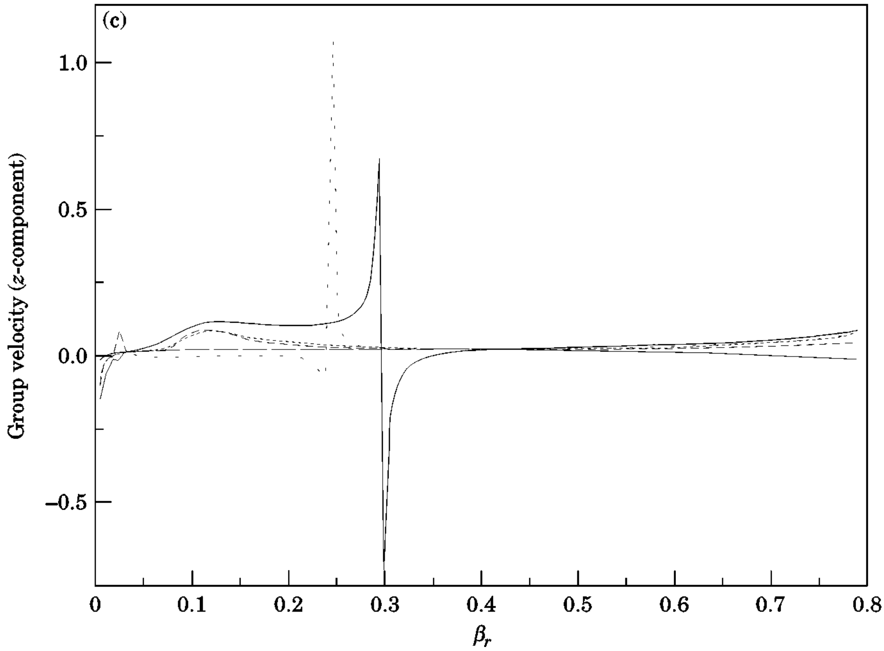

Figure 4. (*Continued*).

exciter. The disturbance waves will propagate predominantly in the streamwise direction, since the streamwise group velocity is much larger than the spanwise component.

As the spanwise wavenumber increases, newer streamwise modes will keep appearing. However, these newer modes have the largest decay rate (by two orders of magnitude higher than the least damped modes) and when the spanwise wavenumber increases, the decay rates of these modes do not change appreciably (results not shown here). The important spanwise wave number range—as indicated in Gaster *et al.* (1994)—is covered in these figures, a noteworthy feature of which is that the plotted quantities do not show large variations for the plotted spanwise modes. For the streamwise wavenumber all the spanwise harmonics produce disturbance waves of very large wavelength and whose appearance would be that of a heaving motion. Also, the plotted streamwise decay rate is lowest for the two predominant modes, and such a wave-train will exhibit very little streamwise decay. Add to this the fact that the generated wave-trains have extremely small phase speeds, and the waves would appear almost static. This has prompted Gaster *et al.* (1994) to state that *for the parameter values used here the motion can be considered to be essentially quasistatic. It is convenient to introduce a periodic perturbation for purely experimental reasons.* The disturbance would propagate mainly in the streamwise direction, as the streamwise group velocity magnitude is orders of magnitude higher than spanwise group velocity. The relative importance of the spanwise modes could only be ascertained by studying the receptivity aspect of the excitation—as was done for a two-dimensional disturbance field in Sengupta *et al.* (1994). Results are obtained for wall excitation at even lower frequencies, and all of them have similar qualitative characteristics.

Finally, we would like to discuss the intensity and phase profiles of the disturbance field. The intensity profile within the boundary layer indicates that if the TS waves are created, then for large y these modes will decay as $\exp[-\sqrt{\alpha^2 + n^2\beta_0^2}y]$ and the largest value of β considered in Figures 3(a,b) and 4(a,b,c) will correspond to $n = 231$. As far as the phase of the disturbance field is concerned, this is determined by α_r and, since the values for the predominant modes are of the order of 10^{-4}, the phase will not change at all with plotted heights—as also seen experimentally in Gaster *et al.* (1994).

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